LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

THIRD SEMESTER – **APRIL 2013**

MT 3501/MT 3500 - ALGEBRA, CALCULUS AND VECTOR ANALYSIS

Date: 29/04/2013 Dept. No. Max. : 100 Marks

Time: 9:00 - 12:00

PART – A

(Answer ALL questions) **(10 × 2 = 20)**

1. Evaluate $\int_0^1 \int_0^1 \int_0^1 x^2$dx dy dz.
2. Show that $\beta(m,n) = \beta(n,m)$.
3. Form a partial differential equation by eliminating the arbitrary constants a and b from z=(x+a)(x+b).
4. State Lagrange's equation.
5. If $\phi(x,y,z) = x^2y + y^2x + z^2$, find $\nabla\phi$.
6. Show that $\bar{F} = (y^2 - z^2 + 3yz - 2x)\vec{i} + (3xz + 2xy)\vec{j} + (3xy - 2xz + 2z)\vec{k}$ is irrotational.
7. Find $L^{-1}\left(\frac{s}{s^2-9}\right)$.
8. Find L (Sin at).
9. Define Euler's function.
10. State Fermat's theorem.

PART - B **(5X8=40)**

(Answer any FIVE questions)

11 Evaluate $\iint xy\, dx\, dy$ over the domain bounded by x-axis, x=2a and the curve $x^2 = 4ay$.

12. Change the order of integration and evaluate $\int_1^2 \int_0^{4-x^2} (x+y)dx\, dy$.

13. Obtain the complete and singular solution of $\frac{z}{pq} = \frac{x}{q} + \frac{y}{p} + \sqrt{pq}$.

14. Solve (mz – ny)p – (nx – lz)q = ly – mx.

15. Find $\int_C F.dr$ where $\bar{F} = (x^2 - y^2)i + 2xyj$ where C is the square bounded by the coordinates axes and the lines x=a and y=b.

16. Find $L\left(\frac{Cos\, 3t - Cos\, 2t}{t}\right)$.

17. Find $L^{-1}\left(\frac{1}{s\,(s^2+a^2)}\right)$.

18. Find the remainder when 2^{1000} is divisible by 17.

PART – C

(Answer any TWO questions) **($2 \times 20 = 40$)**

19. (a) Evaluate $\iiint xyz$ dx dy dz taken through the positive octant of the sphere $x^2 + y^2 + z^2 = a^2$.

(b) Show that β (m, n) = $\frac{\Gamma(n)\Gamma(m)}{\Gamma(m+n)}$.

20. (a) Solve $p^2 + q^2 = z^2(x + y)$

(b) Solve $(3z - 4)p + (4x - 2z)\, q = 2y - 3x$.

21. (a) Show that 8th power of any number is of the form 17p or 17p $\pm$ 1.

(b) Verify Stroke's theorem when $\bar{F} = y\bar{i} + z\,\bar{j} + x\bar{k}$ and the surface is the part of the sphere $x^2 + y^2 + z^2 = 1$ along the XY plane.

22. Using Laplace transform solve $\frac{d^2t}{dt^2} + 6\frac{dy}{dt} + 5t = e^{-2t}$ given that y = 0 and $\frac{dy}{dt}$ = 1 when t= 0.

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